## Geometry <br> Summary of Perimeter and Area Formulas - 2D Shapes

| Shape | Figure | Perimeter | Area |
| :---: | :---: | :---: | :---: |
| Kite |  | $\begin{gathered} P=2 b+2 c \\ b, c=\text { sides } \end{gathered}$ | $\begin{gathered} A=\frac{1}{2}\left(d_{1} d_{2}\right) \\ d_{1}, d_{2}=\text { diagonals } \end{gathered}$ |
| Trapezoid |  | $\begin{gathered} P=b_{1}+b_{2}+c+d \\ b_{1}, b_{2}=\text { bases } \\ c, d=\text { sides } \end{gathered}$ | $\begin{gathered} A=\frac{1}{2}\left(b_{1}+b_{2}\right) h \\ b_{1}, b_{2}=\text { bases } \\ h=\text { height } \end{gathered}$ |
| Parallelogram |  | $\begin{gathered} P=2 b+2 c \\ b, c=\text { sides } \end{gathered}$ | $\begin{gathered} \mathbf{A}=\mathbf{b h} \\ b=\text { base } \\ h=\text { height } \end{gathered}$ |
| Rectangle |  | $\begin{gathered} P=2 b+2 c \\ b, c=\text { sides } \end{gathered}$ | $\begin{gathered} \mathbf{A}=\mathbf{b h} \\ b=\text { base } \\ h=\text { height } \end{gathered}$ |
| Rhombus |  | $\begin{aligned} & P=4 s \\ & s=\text { side } \end{aligned}$ | $\begin{gathered} A=b h=\frac{1}{2}\left(d_{1} d_{2}\right) \\ d_{1}, d_{2}=\text { diagonals } \end{gathered}$ |
| Square |  | $\begin{aligned} & P=4 s \\ & s=s i d e \end{aligned}$ | $\begin{gathered} A=s^{2}=\frac{1}{2}\left(d_{1} d_{2}\right) \\ d_{1}, d_{2}=\text { diagonals } \end{gathered}$ |
| Regular Polygon |  | $\begin{gathered} P=n s \\ n=\text { number of sides } \\ s=\text { side } \end{gathered}$ | $\begin{aligned} A & =\frac{1}{2} a \cdot P \\ a & =\text { apothem } \\ P & =\text { perimeter } \end{aligned}$ |
| Circle | $1$ | $\begin{gathered} C=2 \pi r=\pi d \\ r=\text { radius } \\ d=\text { diameter } \end{gathered}$ | $\begin{gathered} A=\pi r^{2} \\ r=\text { radius } \end{gathered}$ |
| Ellipse |  | $P \approx 2 \pi \sqrt{\frac{1}{2}\left(r_{1}^{2}+r_{2}^{2}\right)}$ <br> $r_{1}=$ major axis radius <br> $r_{2}=$ minor axis radius | $\begin{gathered} A=\pi r_{1} r_{2} \\ r_{1}=\text { major axis radius } \\ r_{2}=\text { minor axis radius } \end{gathered}$ |

## Area Review Worksheet

Name: $\qquad$

1. Find the height of an isosceles trapezoid with an area of 96 and bases equal to 9 and 7 .

There are two options for calculating the area of a trapezoid.
One is to use the standard formula:

$$
A=\frac{1}{2}\left(b_{1}+b_{2}\right) h
$$



The alternative is to calculate the length of the midline and to use the formula:
$A=m h$, where $m$ is the length of the midline, the mean of the base lengths.
The two formulas are identical, except in how you think about them. I find that I can solve a trapezoid area problem quicker in most cases with the midline formula because I can quickly calculate $m$ in my head. For this problem,

$$
\begin{aligned}
& m=\frac{7+9}{2}=8 \text { (typically done in my head) } \\
& A=m h \quad \rightarrow \quad 96=8 h \quad \rightarrow \quad \boldsymbol{h}=\mathbf{1 2} \text { units }
\end{aligned}
$$

2. The diagonals of a kite are 10 and 22 . Find the area of the kite.

The formula for the area of a kite is similar to the formula for the area of a triangle, with the lengths of the two diagonals substituted for the lengths of the base and height.


$$
A=\frac{1}{2} d_{1} d_{2}=\frac{1}{2}(10)(22)=\mathbf{1 1 0} \text { units }^{2}
$$

## 3. Find the area of the parallelogram

The area of a parallelogram is the same as that for a rectangle. The difference is that you need to calculate the height before
 applying the formula: $A=b h$.

In the diagram to the right, the orange has been added to facilitate the required calculation. Note that the height is a leg of a $45^{\circ}-45^{\circ}-90^{\circ}$ triangle, so we have:

$$
\begin{aligned}
& h=\frac{6.4}{\sqrt{2}} \\
& A=b h=12.8 \cdot\left(\frac{6.4}{\sqrt{2}}\right)=57.93 \text { units }^{2}
\end{aligned}
$$

## Use the diagram to the right for \#4-5.

4. Find the apothem of the regular hexagon with a side length of 12 .

In the diagram to the right, the orange has been added to facilitate the required calculation. The apothem splits the bottom side in half, i.e., into two segments of length 6.

Each interior angle in a regular hexagon is $120^{\circ}$, so half of that is

$60^{\circ}$. By adding the orange segment, we get a $30^{\circ}-60^{\circ}-90^{\circ}$ triangle, so we can calculate:

$$
a=6 \cdot \sqrt{3}=6 \sqrt{3} \text { units }
$$

5. Find the area of the regular hexagon.

The perimeter of this regular hexagon is: $P=(6$ sides $) \cdot(12$ units long $)=72$ units
The area of a regular polygon is:

$$
A=\frac{1}{2} a P=\frac{1}{2}(6 \sqrt{3}) \cdot 72=216 \sqrt{3} \text { units }^{2}
$$

6. Find the area of an equilateral triangle with a side length 9 .

We need the base and the height of this triangle. We are given the base has length 9 .

To get the height, draw an altitude from the top to the opposite side and notice we end up with a $30^{\circ}-60^{\circ}-90^{\circ}$ triangle, so we can calculate the height as:


$$
h=\frac{\text { side }}{2} \cdot \sqrt{3}=\frac{9}{2} \sqrt{3}
$$

Then, calculate the area with the standard triangle formula for area:

$$
A=\frac{1}{2} b h=\frac{1}{2} \cdot 9 \cdot \frac{9}{2} \sqrt{3}=\frac{81 \sqrt{3}}{4} \text { units }^{2}
$$

If you like memorizing formulas, the general formula for the area of an equilateral triangle can be calculated using the same technique that was used in this problem. Let $s$ be the length of a side of the triangle, then $\frac{s}{2} \sqrt{3}$ is the length of the height. Area then, is:

$$
A=\frac{1}{2} b h=\frac{1}{2} \cdot s \cdot \frac{s}{2} \sqrt{3}=\frac{\sqrt{3}}{4} s^{2}
$$

7. Find the area of the annulus shown.

An annulus is the area between two circles, so its area is the difference of the areas of the two circles:

$$
\begin{aligned}
& A_{\text {large }}=\pi r_{\text {large }}^{2}=\pi \cdot(8+2)^{2}=100 \pi \\
& A_{\text {small }}=\pi r_{\text {small }}^{2}=\pi \cdot 8^{2}=64 \pi \\
& A_{\text {annulus }}=A_{\text {large }}-A_{\text {small }}=100 \pi-64 \pi=36 \pi \text { units }^{2}
\end{aligned}
$$



We need the lengths of the diagonals of the kite.
The vertical diagonal has length $d_{1}=8+8=16$.
To find the horizontal diagonal, we need the help of Pythagoras.

$$
\begin{aligned}
& x^{2}+8^{2}=17^{2} \quad \rightarrow \quad x=15 \\
& d_{2}=15+6=21
\end{aligned}
$$

Finally, we have:

$$
A=\frac{1}{2} d_{1} d_{2}=\frac{1}{2}(16)(21)=168 \text { units }^{2}
$$

9. Find the area of an equilateral triangle with an altitude of $10 \sqrt{3}$.

In this problem, we are given $h$, and we need to find $b$.
We draw an altitude from the top of the triangle to the base, creating a pair of congruent interior triangles. Since we then have $30^{\circ}-60^{\circ}-90^{\circ}$ triangles, we determine that the base on one of them is 10 . The
 whole base, then is $2 \cdot 10=20$. Finally,

$$
A=\frac{1}{2} b h=\frac{1}{2}(10+10)(10 \sqrt{3})=100 \sqrt{3} \text { units }^{2}
$$

If you like memorizing formulas, the general formula for the area of an equilateral triangle can be calculated using the same technique that was used in this problem. Let $h$ be the length of an altitude of the triangle, then $\frac{2 h}{\sqrt{3}}$ is the length of the base. Area then, is:

$$
\boldsymbol{A}=\frac{1}{2} b h=\frac{1}{2} \cdot\left(\frac{2 h}{\sqrt{3}}\right) \cdot h=\frac{1}{\sqrt{3}} \boldsymbol{h}^{2}
$$

10. Find area of the sector BAC.


The sector shown is $60^{\circ}$ of the total $360^{\circ}$ around the circle.
Sector area $=\frac{60}{360} \cdot \pi \cdot 10^{2}=\frac{\mathbf{5 0 \pi}}{3}$ units $^{2}$.
11. Find the area of triangle $A B C$


This is an equilateral triangle with sides of length 10. Let's use the formula for the area of an equilateral triangle:

$$
A=\frac{\sqrt{3}}{4} s^{2}=\frac{\sqrt{3}}{4}(10)^{2}=25 \sqrt{3} \text { units }^{2}
$$

12. Find the area of segment $\overline{B C}$


The sector shown measures $60^{\circ}$ (the same as the central angle it subtends). So, we can use the results of Problems 10 and 11 above.

Shaded area $=$ sector area - triangle area.
Shaded area $=\frac{50 \pi}{3}-25 \sqrt{3}$ units $^{2}$
13. Find the area of a circle whose circumference is $36 \pi$

$$
\begin{aligned}
& C=2 \pi r=36 \pi \quad \rightarrow \quad r=18 \\
& A=\pi r^{2}=\pi \cdot\left(18^{2}\right)=324 \pi \text { units }^{2}
\end{aligned}
$$

14. If the area of a square is 60 , find the perimeter.

$$
\begin{aligned}
& A=s^{2}=60 \quad \rightarrow \quad s=\sqrt{60}=2 \sqrt{15} \\
& P=4 s=4 \cdot 2 \sqrt{15}=\mathbf{8} \sqrt{\mathbf{1 5}} \text { units }
\end{aligned}
$$

## 15. Find the area of the isosceles trapezoid.

The green and orange have been added to the diagram to help us solve this problem. Let's use the midline formula: $m$ is the mean of 12 and 16 , so $m=14$.

To find $h$, we create the triangle formed by dropping a segment from the top left vertex of the isosceles trapezoid. This gives us a $30^{\circ}-60^{\circ}-90^{\circ}$ triangle.

Notice in the diagram, that $2 a+12=16$, so $a=2$.
Then, $h=2 \sqrt{3}$ because of the $30^{\circ}-60^{\circ}-90^{\circ}$ triangle.

$$
A=m h=14 \cdot 2 \sqrt{3}=28 \sqrt{3} \text { units }^{2}
$$


16. Find the area of the shaded region if the radius of the circle is 7 and the triangle is equilateral.

$7 \sqrt{3}$

The orange items have been added to the diagram to help us solve the problem. To calculate the shaded area , we can use:

$$
\text { shaded area }=\text { large triangle area }- \text { circle area }
$$

The central angle identified is $60^{\circ}$ because 6 of them would be required to go around an entire circle, and $360^{\circ} \div 6=60^{\circ}$.

The small triangle (bottom left of the diagram) has a short side of length 7, so we can fill out the lengths of the other sides based on the fact that we have a $30^{\circ}-60^{\circ}-90^{\circ}$ triangle. The length of a side of the large triangle, then, is: $2 \cdot 7 \sqrt{3}=14 \sqrt{3}$.

Finally, the areas we want are:

$$
\begin{aligned}
& A_{\text {large triangle }}=\frac{\sqrt{3}}{4} s^{2}=\frac{\sqrt{3}}{4}(14 \sqrt{3})^{2}=147 \sqrt{3} \\
& A_{\text {circle }}=\pi r^{2}=\pi(7)^{2}=49 \pi \\
& \text { shaded area }=147 \sqrt{3}-49 \pi \text { units }^{2}
\end{aligned}
$$

17. Find the area of the shaded region if the radius of each circle is 6 .


12

If the radii of the circles are 6 , the side of the square is: $2 \cdot 6=12$.
shaded area $=$ square area - circle area
square area $=12^{2}=144$
The two half circles make a whole circle, so
circle area $=\pi r^{2}=\pi(6)^{2}=36 \pi$
shaded area $=$ square area - circle area $=144-36 \pi$ units $^{2}$
18. If $\triangle A B C \sim \triangle D E F$, find the ratio of the area of $\triangle A B C$ to $\triangle D E F$.


The ratio of the areas is the square of the ratio of the linear measures.

$$
r=\frac{\triangle A B C \text { area }}{\triangle D E F \text { area }}=\left(\frac{15}{20}\right)^{2}=\left(\frac{3}{4}\right)^{2}=\frac{9}{16} \text { or } 9: 16
$$

19. If the ratio of area for two similar polygons is 16:49, find the ratio of their corresponding sides.

The ratio of the areas is the square of the ratio of the linear measures. So, the ratio of linear measures is the square root of the ratio of the areas.

$$
r=\sqrt{\frac{16}{49}}=\frac{4}{7} \text { or } 4: 7
$$

20. Find the ratio of the area of $\triangle A B C$ to $\triangle A C D$.


Base lengths and heights are the same for these two triangles; therefore, their areas are the same. The ratio is:

1 or 1:1
21. If a regular hexagon is inscribed in a circle with a radius of 7 , find the area of the shaded region.

We are looking for the shaded area.
If the radius of the circle is 7 , a side of the hexagon is also 7 .
Hexagons are full of $60^{\circ}$ angles, which is helpful. This allows us to calculate the lengths of the two remaining sides of the orange triangle as shown in the diagram. Then:
circle area $=\pi r^{2}=\pi(7)^{2}=49 \pi$
hexagon area $=\frac{1}{2}$ ap $=\frac{1}{2}\left(\frac{7}{2} \sqrt{3}\right)(6 \cdot 7)=\frac{147}{2} \sqrt{3}$
shaded area $=$ circle area - hexagon area
shaded area $=$ circle area - hexagon area $=49 \pi-\frac{147}{2} \sqrt{3}$ units $^{2}$
22. A succession of squares is formed by joining the midpoints of each side of each square.

If the length of each side of the large square is 20 , find the area of the shaded square.
Notice that we are able to create a $45^{\circ}-45^{\circ}-90^{\circ}$ triangle in the upper right corner of the diagram.

Working in from the outer square to the next inner square, we see that the side lengths of the squares shrink by a factor of $\sqrt{2}$.
Since the side lengths shrink by a factor of $\sqrt{2}$, the areas of
successive squares must shrink by a factor of $(\sqrt{2})^{2}=2$.


The outer square has an area of: $A=20^{2}=400$ units $^{2}$.
The shaded square is three squares in from the outer square, so its area must be:

$$
A=400 \cdot\left(\frac{1}{2}\right)^{3}=50 \text { units }^{2}
$$

23. A trapezoid has a height of 13 meters, a base length of 11 meters, and an area of 92 square meters. What is the length of the other base.

We have $A=92, h=13, b_{1}=11$. We are asked for the value of $b_{2}$.
It is easiest to work with the basic trapezoid area formula for this problem.

$$
\begin{aligned}
& A=\frac{1}{2}\left(b_{1}+b_{2}\right) h \quad \rightarrow \quad 92=\frac{1}{2}\left(11+b_{2}\right) \cdot 13 \\
& \frac{184}{13}=11+b_{2} \quad \rightarrow \quad \boldsymbol{b}_{2}=\left(\frac{184}{13}\right)-11=\frac{41}{13} \approx 3.15 \mathrm{~m}
\end{aligned}
$$

